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# AS Level Mathematics A

**H230/02** Pure Mathematics and Mechanics
Sample Question Paper

## Date – Morning/Afternoon

Version 2.1

Time allowed: 1 hour 30 minutes

**You must have:**
- Printed Answer Booklet

**You may use:**
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

**INFORMATION**

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

# Formulae
# AS Level Mathematics A (H230)

## Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \ldots\ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \ldots\ + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

## Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

## Standard deviation

$$\sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f\left(x-\bar{x}\right)^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$$

## The binomial distribution

If $X \sim \mathrm{B}(n,\ p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of $X$ is $np$, variance of $X$ is $np(1-p)$

## Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

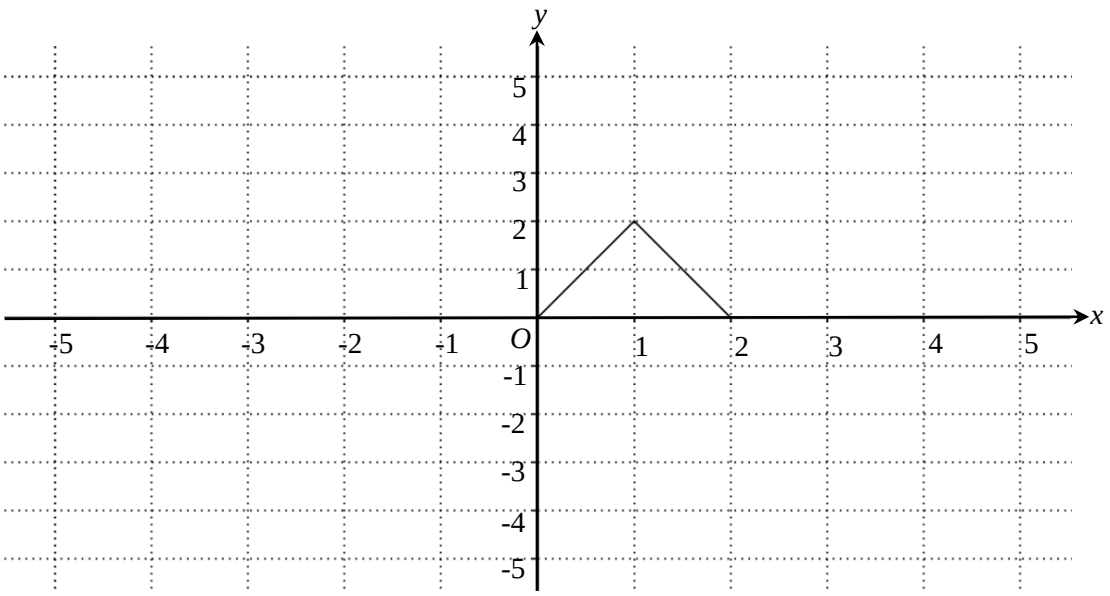
$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

## Section A: Pure Mathematics

Answer **all** the questions

**1** **(a)** The diagram below shows the graph of $y = \mathrm{f}(x)$.




**(i)** On the diagram in the Printed Answer Booklet draw the graph of $y = \mathrm{f}(x+3)$. **[2]**

**(ii)** Describe fully the transformation which transforms the graph of $y = \mathrm{f}(x)$ to the graph of $y = -\mathrm{f}(x)$. **[1]**

**(b)** The point (2, 3) lies on the graph of $y = \mathrm{g}(x)$.

State the coordinates of its image when $y = \mathrm{g}(x)$ is transformed to

**(i)** $y = 4\mathrm{g}(x)$ **[1]**

**(ii)** $y = \mathrm{g}(4x)$. **[1]**

**2** **In this question you must show detailed reasoning.**

Solve the equation $2\cos^2 x = 2 - \sin x$ for $0^\circ \leq x \leq 180^\circ$. **[5]**

**3** The number of members of a social networking site is modelled by $m = 150e^{2t}$, where $m$ is the number of members and $t$ is time in weeks after the launch of the site.

**(a)** State what this model implies about the relationship between $m$ and the rate of change of $m$. **[2]**

**(b)** What is the significance of the integer 150 in the model? **[1]**

**(c)** Find the week in which the model predicts that the number of members first exceeds 60 000. **[3]**

**(d)** The social networking site only expects to attract 60 000 members.

Suggest how the model could be refined to take account of this. **[1]**

**4** The points $A$, $B$ and $C$ have position vectors $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ respectively.

$M$ is the midpoint of $BC$.

**(a)** Find the position vector of the point $D$ such that $\overrightarrow{BC} = \overrightarrow{AD}$. **[3]**

**(b)** Find the magnitude of $\overrightarrow{AM}$. **[3]**

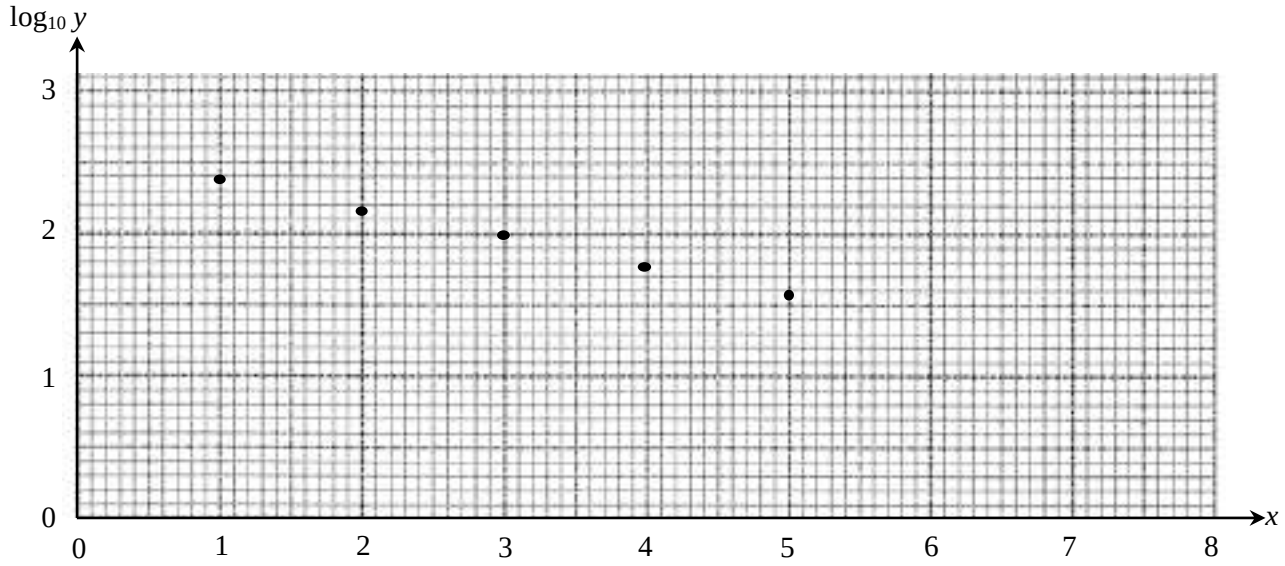
**5** A doctors' surgery starts a campaign to reduce missed appointments.
The number of missed appointments for each of the first five weeks after the start of the campaign is shown below.

| Number of weeks after the start ($x$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of missed appointments ($y$) | 235 | 149 | 99 | 59 | 38 |

This data could be modelled by an equation of the form $y = pq^x$ where $p$ and $q$ are constants.

**(a)** Show that this relationship may be expressed in the form $\log_{10} y = mx + c$, expressing $m$ and $c$ in terms of $p$ and/or $q$. **[2]**

The diagram below shows $\log_{10} y$ plotted against $x$, for the given data.




**(b)** Estimate the values of $p$ and $q$. **[3]**

**(c)** Use the model to predict when the number of missed appointments will fall below 20.

Explain why this answer may not be reliable. **[2]**

**6** **(a)** A student suggests that, for any prime number between 20 and 40, when its digits are squared and then added, the sum is an odd number.

For example, 23 has digits 2 and 3 which gives $2^2 + 3^2 = 13$, which is odd.

Show by counter example that this suggestion is false. **[2]**

**(b)** Prove that the sum of the squares of any three consecutive positive integers cannot be divided by 3. **[3]**

**7** Differentiate $\mathrm{f}(x) = x^4$ from first principles. **[5]**

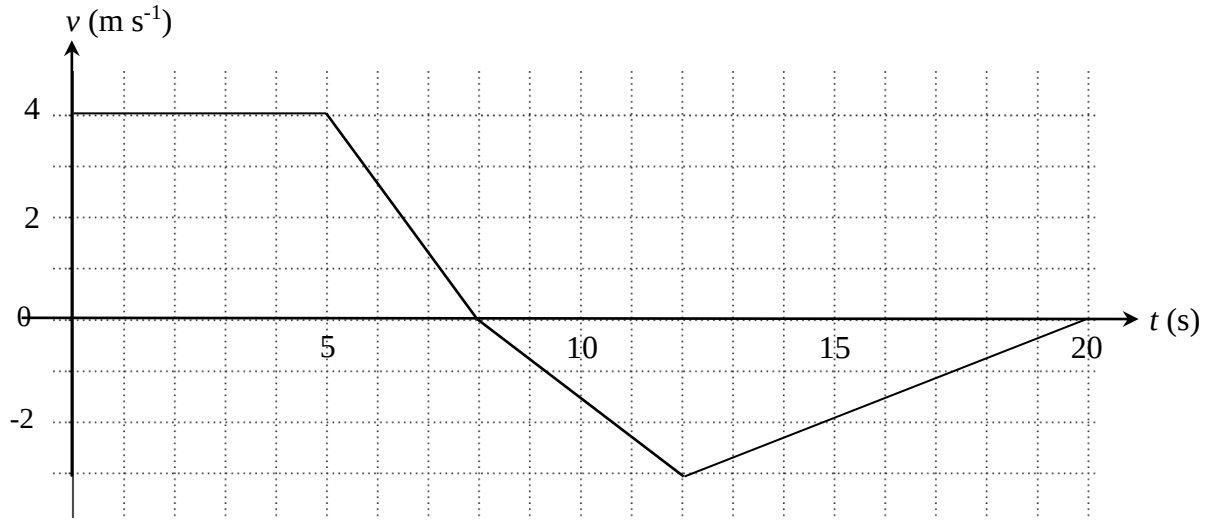
**8** A curve has equation $y = kx^{\frac{3}{2}}$ where $k$ is a constant.
The point $P$ on the curve has $x$-coordinate 4.
The normal to the curve at $P$ is parallel to the line $2x + 3y = 0$ and meets the $x$-axis at the point $Q$.
The line $PQ$ is the radius of a circle centre $P$.

Show that $k = \frac{1}{2}$.
Find the equation of the circle. **[10]**

## Section B: Mechanics

Answer **all** the questions

**9** The diagram below shows the velocity-time graph of a car moving along a straight road, where $v$ m s$^{-1}$ is the velocity of the car at time $t$ s after it passes through the point $A$.




**(a)** Calculate the acceleration of the car at $t = 6$. **[2]**

**(b)** Jasmit says "The distance travelled by the car during the first 20 seconds of the car's motion is more than five times its displacement from $A$ after the first 20 seconds of the car's motion".

Give evidence to support Jasmit's statement. **[3]**

**10** A student is attempting to model the flight of a boomerang.
She throws the boomerang from a fixed point $O$ and catches it when it returns to $O$.

She suggests the model for the displacement, $s$ metres, after $t$ seconds is given by

$s = 9t^2 - \frac{3}{2}t^3,\ 0 \leq t \leq 6.$

For this model,

**(a)** determine what happens at $t = 6$, **[2]**

**(b)** find the greatest displacement of the boomerang from $O$, **[4]**

**(c)** find the velocity of the boomerang 1 second before the student catches it, **[2]**

**(d)** find the acceleration of the boomerang 1 second before the student catches it. **[2]**

**11** In this question the unit vectors **i** and **j** are in the directions east and north respectively.

Distance is measured in metres and time in seconds.

A ship of mass 100 000 kg is being towed by two tug boats.

- The cables attaching each tug to the ship are horizontal.
- One tug produces a force of $(350\mathbf{i} + 400\mathbf{j})$ N.
- The other tug produces a force of $(250\mathbf{i} - 400\mathbf{j})$ N.
- The total resistance to motion is 200 N.
- At the instant when the tugs begin to tow the ship, it is moving east at a speed of 1.5 m $s^{-t}$.

**(a)** Explain why the ship continues to move directly east. **[2]**

**(b)** Find the acceleration of the ship. **[2]**

**(c)** Find the time which the ship takes to move 400 m while it is being towed.

Find its speed after moving that distance. **[6]**

**END OF QUESTION PAPER**